Algebraic Number Theory: Presentations Guidelines

There are many topics in Algebraic Number Theory that we will not cover in this course. Choose one of those topics to study independently and present to the class in November.

Presentations will be 20 minutes, plus 5 minutes for questions and transition to the next speaker, and will take place in class **between November 14 and November 21**. Presentations should not just be a superficial overview of a topic, but should include at least one substantive proof and one substantive example. It is recommended, but not required, that your presentation be slide talk using Beamer. (If you feel strongly that your topic would be better communicated in a different format, please do so.)

Optionally, you may make work with another student to coordinate topics—in other words, one person's talk would contain background material used in the other person's talk. Please let me know if you are planning to do this. Additionally, if two people chose closely related topics, I may ask you to communicate with each other to avoid repetition.

Talk titles and abstracts are due on **October 31**. Abstracts need only be a few sentences long. If you want feedback on your choice of topic, please ask when you turn in your abstract or earlier.

I will send out a survey asking for your preference about what day to speak. The options are:

- Tuesday, November 14, in person
- Thursday, November 16, in person
- Tuesday, November 21, **on Zoom** (Thanksgiving week)

Classes with presentations will be considered normal classes, including the Zoom class on November 21, and you are expected to attend and engage with the other presentations by asking questions when appropriate.

The following topics are to be covered (or already covered) in the course, so they are **not allowed** for presentations, unless the presentation goes beyond the course material in a substantial way.

- DVRs and Dedekind domains
- Basics of orders in number fields
- Discriminant and different
- Finiteness of the class group
- Dirichlet unit theorem
- Splitting of primes in extensions
- Dedekind zeta functions
- Regulator and class number formula
- Statement of main theorems of class field theory

It is also recommended that you avoid topics that were covered in Dr. Tu's class in Spring 2023 (quadratic/cubic/quartic reciprocity, Gauss and Jacobi sums, zeta functions of varieties, elliptic curves, etc.) and topics that will likely be covered in the Dr. Allen and Dr. Tu's class in Spring 2024 (modular forms and closely related topics).

Some sources of possible topics are the following books and notes.

- Neukirch, *Algebraic Number Theory*
- Cohen, *A Course in Computational Algebraic Number Theory*
- Crandall and Pomerance, *Prime Numbers: A Computational Perspective*
- Ireland and Rosen, *A Classical Introduction to Modern Number Theory*
- Marcus, *Number Fields*
- Miller and Takloo-Bighash, *An Invitation to Modern Number Theory*

- Serre, *Local Fields*
- Keith Conrad's "blurbs" (online)
- Notes from past Arizona Winter Schools (online, arithmetic geometry topics)

A very incomplete list of possible topics, of varying difficulties, follows. It is recommended that topics be narrow enough that they can be effectively communicated in 20 minutes; some of these will need to be narrowed. Topics with a star (*) require some substantial mathematical background not provided by this course or its prerequisites. Many topics overlap with each other.

- Adeles and ideles
- Algebraic K-groups of number fields (*)
- Algorithms for computing rings of integers, unit groups, etc.
- The Baker–Stark–Heegner theorem (*)
- Brauer groups of local and/or global fields (*)
- Chabauty's method or other techniques for enumerating rational points on curves (*)
- Chevalley's unit theorem
- Continued fractions and units of real quadratic fields
- Eisenstein recpiprocity and/or the Stickleberger relation
- Cyclotomic units
- The Delone–Faddeev correspondence, the Davenport–Heilbronn theorem, and/or Bhargava's "higher composition laws"
- "The elementary theory of finite fields" (James Ax, 1968; first-order logic) (*)
- Galois cohomology and Hilbert's theorem 90
- Gauss composition of quadratic forms and equivalence with the class group
- Hecke characters and Hecke L-functions, and/or Artin L-functions (attached to representations of the Galois group)
- "A Kronecker limit formula for real quadratic fields" (Don Zagier, 1975) (includes several interesting theorems that could each be the topic of a talk)
- Kronecker–Weber theorem (abelian extensions of $\mathbb{Q}$)
- Kummer's congruence, regular primes, and/or the p-adic Riemann zeta function
- Kummer theory (concerning cyclic and abelian extensions)
- Lattice-based cryptography (e.g., NTRU, RLWE)
- Mahler measure
- The number field sieve algorithm for factoring
- Odlyzko bounds (improvement on Minkowski's bound)
- Orders in quaternion algebras
- Ostrowski's theorem
- The p-adic numbers, Hensel's lemma, p-adic fields, and/or topics in valuation theory
- Reimann–Roch theory and/or Arakelov theory (Neukirch Chapter III)
- Ring class groups and/or conductor ideals of orders
- Roth's theorem (in Diophantine approximation) (*)
- Shintani's unit theorem
- Tate's thesis

If you choose, you may give a shortened version (5-10 minutes) of your presentation as a lightning talk at BARD 3 (November 3). You may sign up to do so when you register for BARD 3.

Time allowing, I may choose to cover in class a few of the above topics that are *not* chosen by students.